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Comparison of various models for piezoelectric receivers in wireless acoustic power transfer

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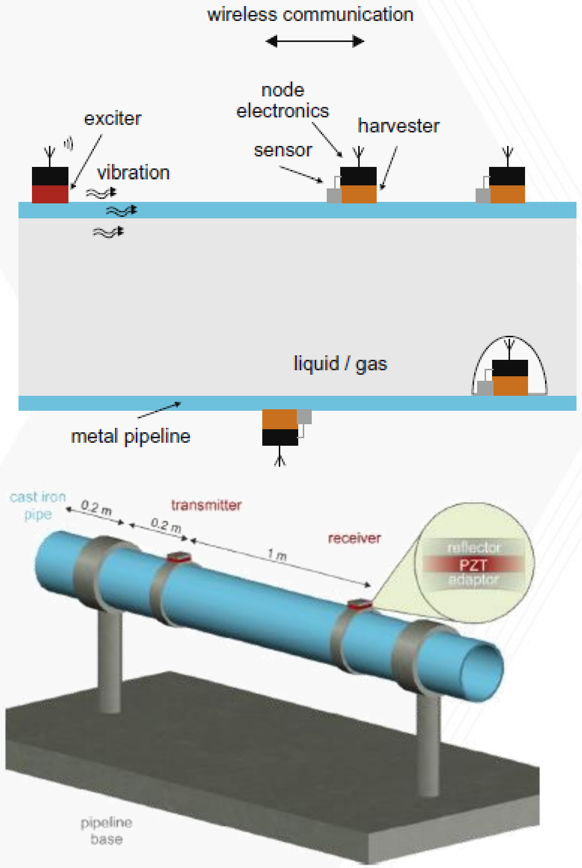
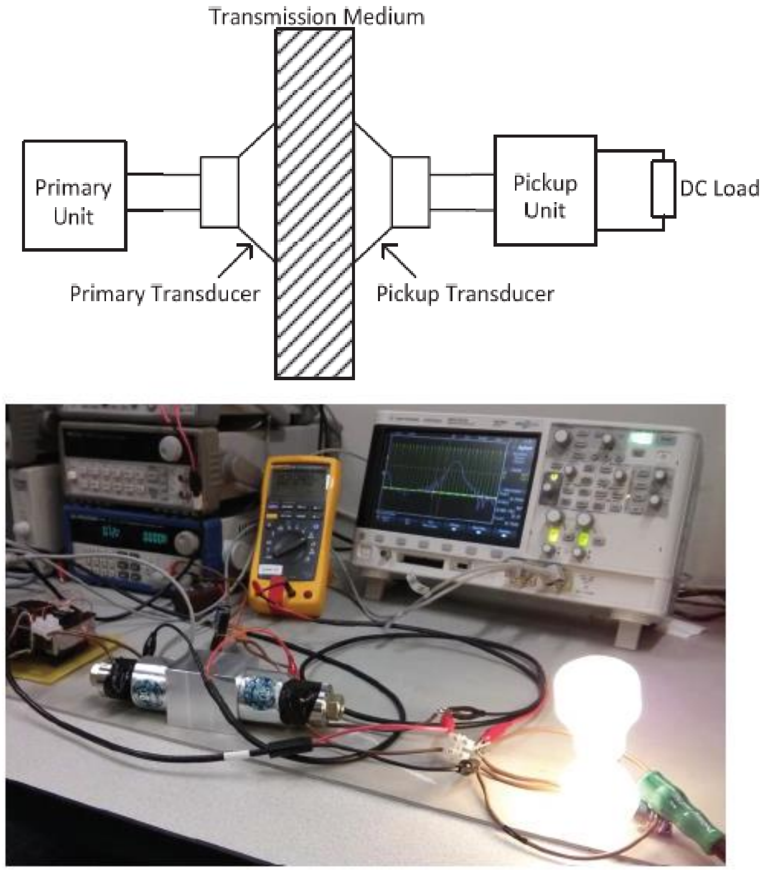
Acoustic power transfer: Example applications

Power transmission to pipeline monitoring sensors



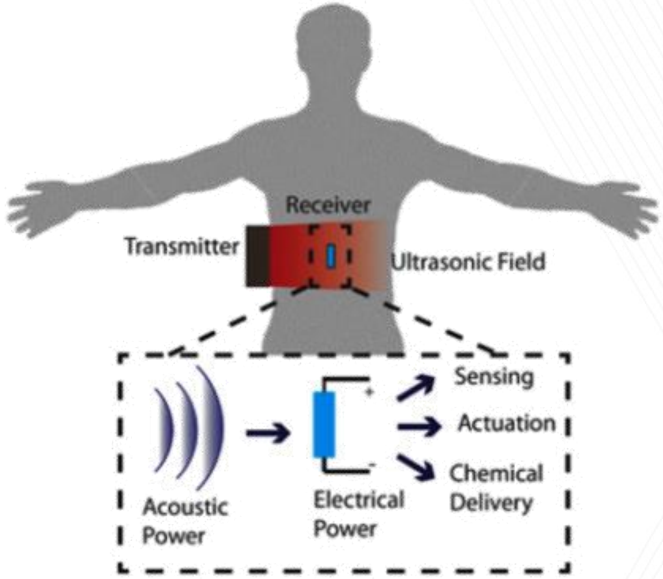
Kiziroglou et al. (2017)

Power transmission through metallic walls

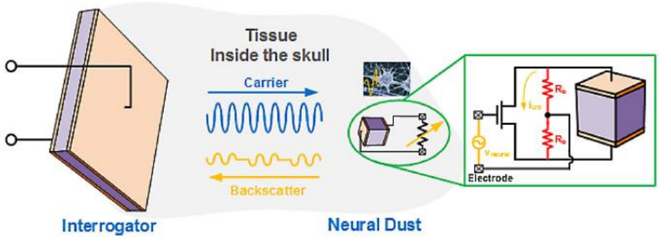


Leung et al. (2014)

Biomedical implants



Song et al. (2015)



Seo et al. (2015)

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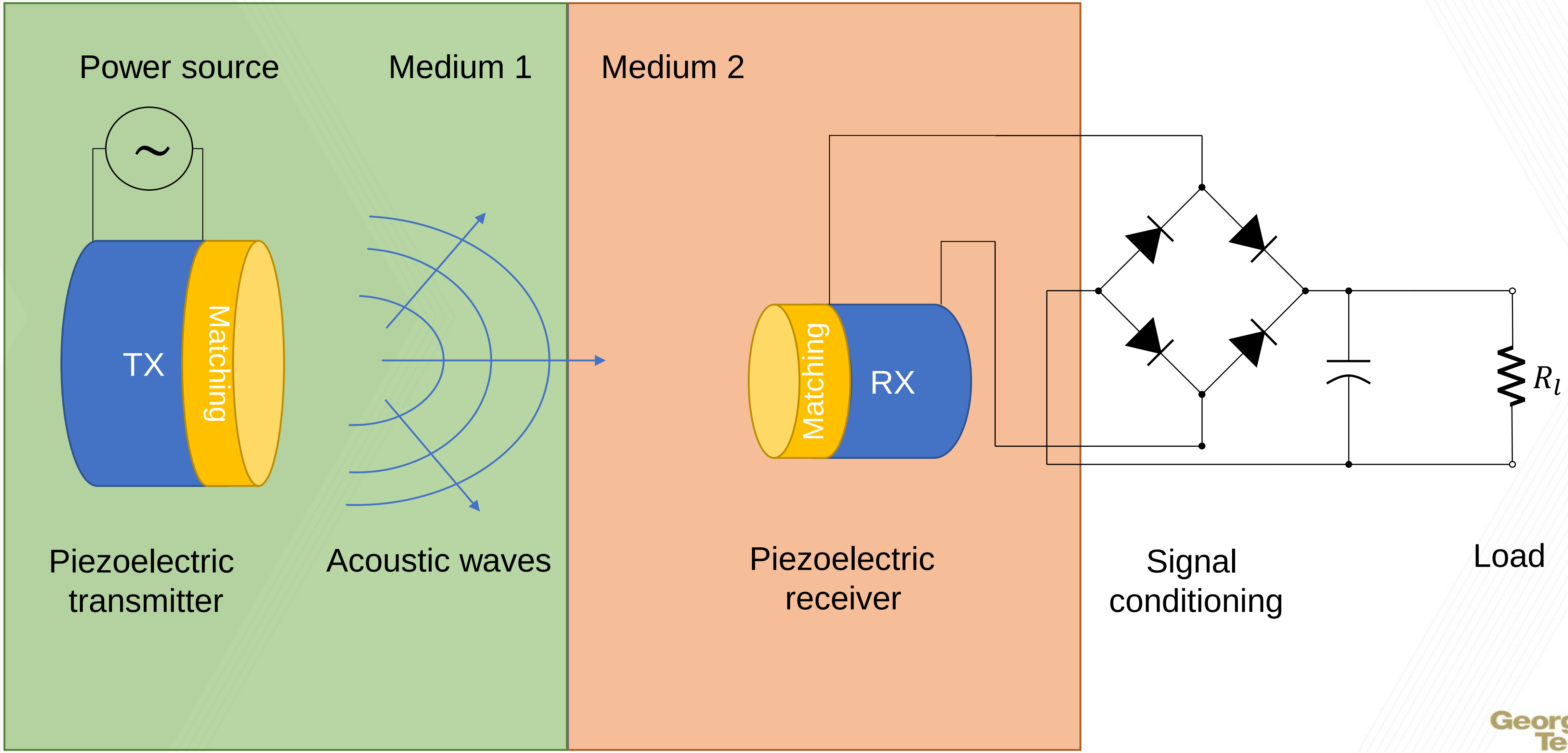
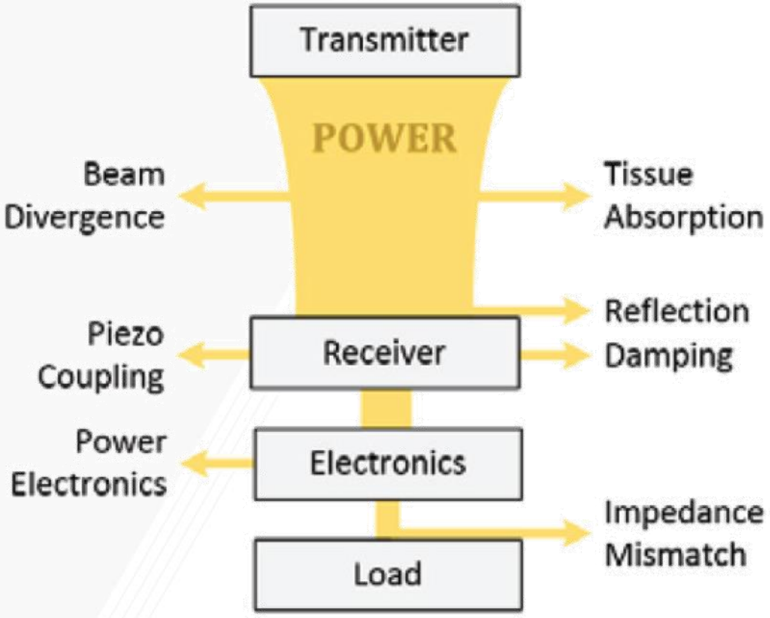
Acoustic power transfer: Basic system components

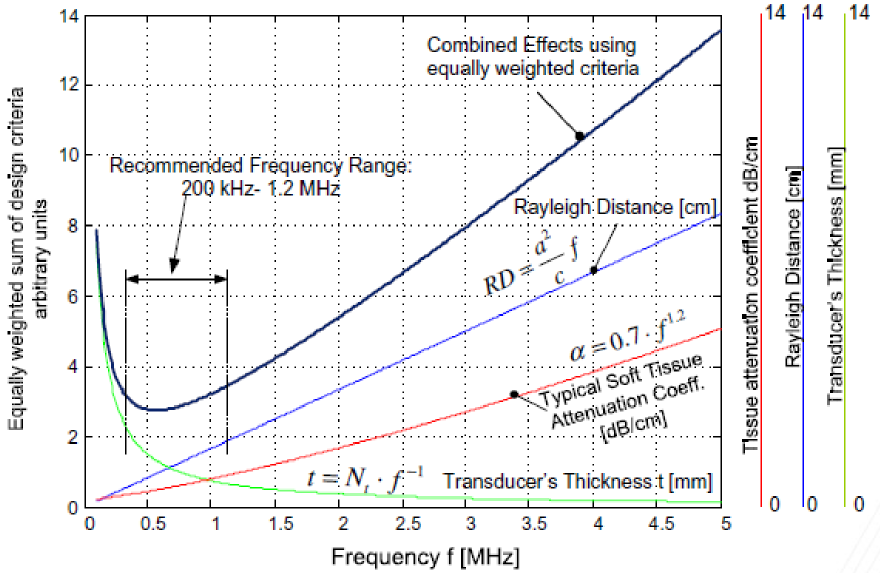


Research challenges

- Power losses due to:
  - Reflections (electrical and acoustic impedance mismatch)
  - Beam divergence, diffraction, etc.
  - Medium absorption
- Strong multi-physics coupling (modeling challenges)



Christnen and Roundy (2015)



Ozeri and Shmilovitz (2010)

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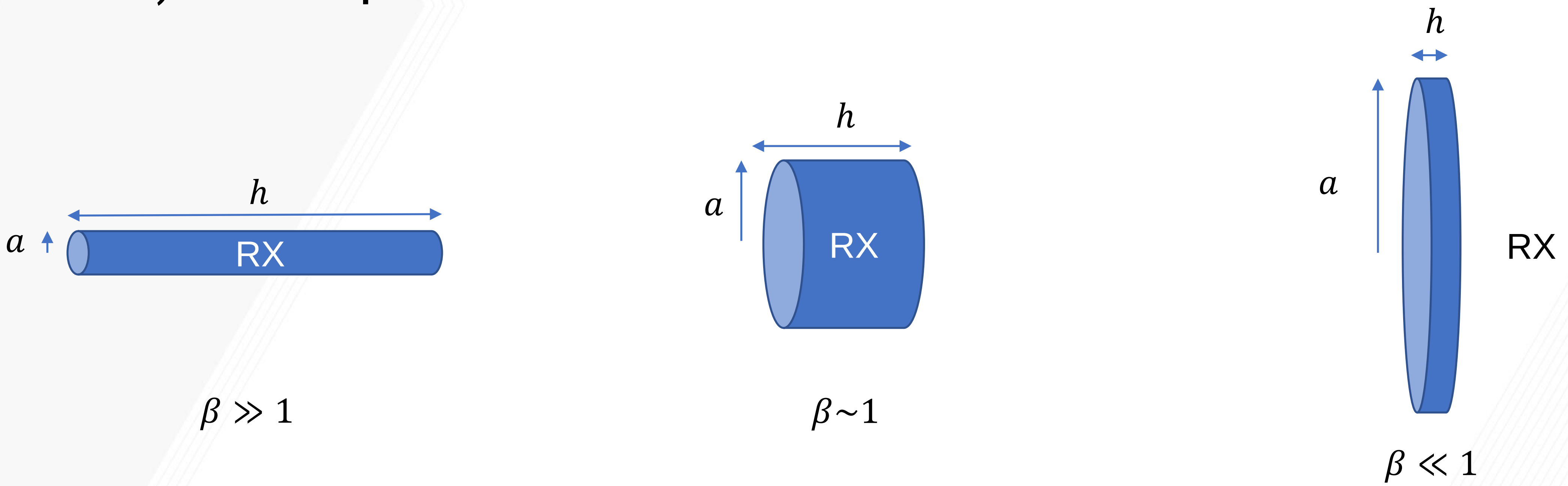
Work Objective

- Investigate the analytical models for modeling fluid loaded 33-mode bulk piezoelectric transducers.

- Determine the effect of changing the height-to-radius (aspect ratio $\beta = {}^{h}/_{a}$) on the performance of PZT receivers.



Constitutive equations for a 33-mode transducer

- Piezoelectric equations:

$$\sigma = \boldsymbol{C}^E S - e^T E$$

$$D = \boldsymbol{e} S + \boldsymbol{\epsilon}^S E$$

- For a thickness-poled cylinder:

$$\sigma_{\mathrm{r}} = C_{11} s_r + C_{12} s_\theta + C_{13} s_{\mathrm{z}} - e_{31} E_{\mathrm{z}}$$

$$\sigma_\theta = C_{12} s_{\mathrm{r}} + C_{22} s_\theta + C_{13} s_{\mathrm{z}} - e_{31} E_{\mathrm{z}}$$

$$\sigma_{\mathrm{z}} = C_{13} s_r + C_{13} s_\theta + C_{33} s_{\mathrm{z}} - e_{33} E_{\mathrm{z}}$$

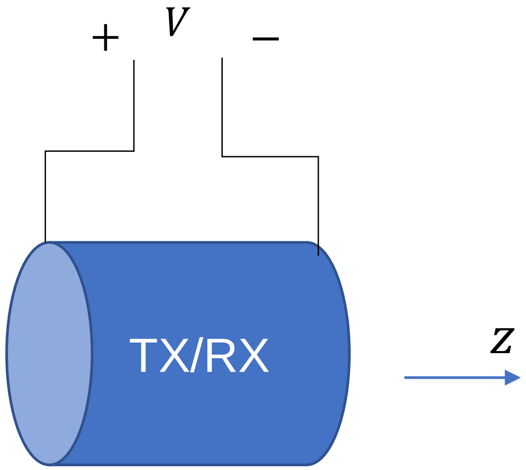
$$\sigma_{\mathrm{rz}} = C_{44} s_{rz}$$

$$\sigma_{\theta\mathrm{z}} = C_{44} s_{\theta z}$$

$$\sigma_{\mathrm{r}\theta} = ((C_{11} - C_{12})/2)\ s_{r\theta}$$

$$\mathrm{D_r} = \mathrm{D}_\theta = 0$$

$$D_{\mathrm{z}} = e_{31} s_{\mathrm{r}} + e_{31} s_\theta + e_{33} s_{\mathrm{z}} + \epsilon_{33} E_{\mathrm{z}}$$



Various theories for analytic modeling

Thin rod (classical)

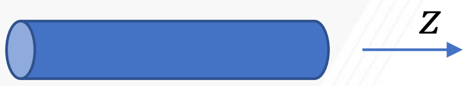
Basis for equivalent electrical circuits (Mason, KLM, ….,etc)

Assumptions:

All lateral stresses and shear **stresses** negligible

$$\sigma_r = \sigma_\theta = \sigma_{rz} = \sigma_{\theta z} = \sigma_{r\theta} = 0$$

$$u_z = u(z,t)$$



Rayleigh

All shear **stresses** negligible:

$$\sigma_{rz} = \sigma_{\theta z} = \sigma_{r\theta} = 0$$

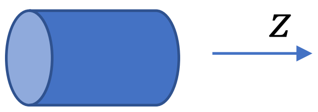
Bishop

Add shear stresses to Rayleigh

Include the effects of lateral inertia:

$$u_z = u(z,t)$$

$$u_r = -\nu\, r \frac{du_z}{dz}$$



Thin plate

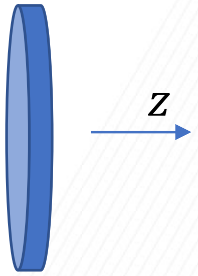
Basis for equivalent electrical circuits (Mason, KLM, ….,etc)

Assumptions:

All lateral and shear **strains** are negligible:

$$s_r = s_\theta = s_{rz} = s_{\theta z} = s_{r\theta} = 0$$

$$u_z = u(z,t)$$



Solution approach

Piezoelectric equations
+
Simplifying assumptions

→ Hamilton's principle →

- Electromechanical EOMs
- Mechanical BCs (continuity equations)
- Electric BCs

$$\int_{t_1}^{t_2} \delta\,(T - U + W_e + W_{nc})\,dt = 0$$

$$W_e = \frac{1}{2}\int_V E_3 D_3 dV$$

$$W_{nc} = \int_S (\bar{t}_r u_r + \bar{t}_\theta u_\theta + \bar{t}_z u_z - \bar{q}\phi) dS$$

For a thin rod:

Electromechanical EOMs:

$$\rho u^{(0,2)}(z,t) - \bar{\mathrm{C}} u^{(2,0)}(z,t) + \bar{\mathrm{e}}\phi^{(2,0)}(z,t) = 0$$

$$\bar{e} u^{(2,0)}(z,t) - \bar{\epsilon}\phi^{(2,0)}(z,t) = 0$$

Mechanical BCs:

$$-\mathrm{A_p}\left(\bar{\mathrm{C}}\, u^{(1,0)}(0,t) + \bar{\mathrm{e}}\,\phi^{(1,0)}(0,t)\right) + P = 0$$

Electric BCs:

$$\mathrm{A_p}\left(\bar{\mathrm{e}} u^{(1,0)}(0,t) - \bar{\epsilon}\phi^{(1,0)}(0,t)\right) - Q = 0$$

Coupling fluid acoustics with transducer dynamics

- Assume wave solution:

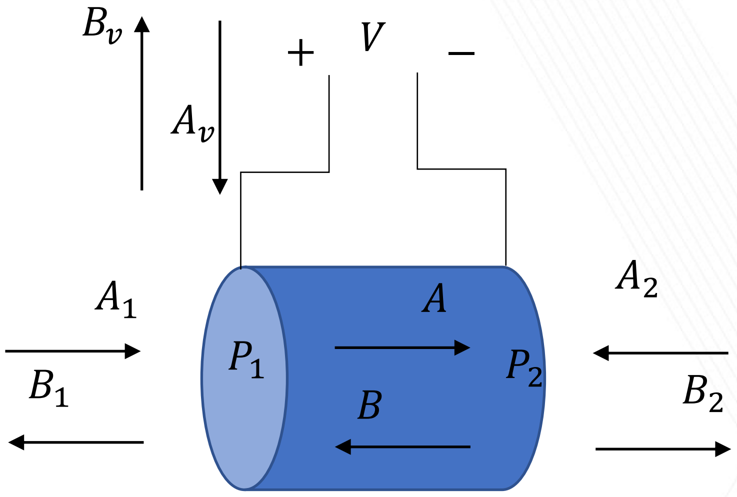
$$P_1(\mathrm{t}) = A_1 e^{j(\omega t - k z_1)} + B_1 e^{j(\omega t + k z_1)}$$

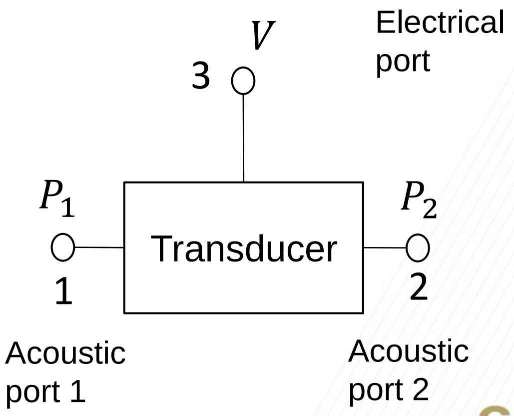
$$P_2(t) = A_2 e^{j(\omega t - k z_2)} + B_2 e^{j(\omega t + k z_2)}$$

$$V(t) = (A_v + B_v) e^{j\omega t}$$

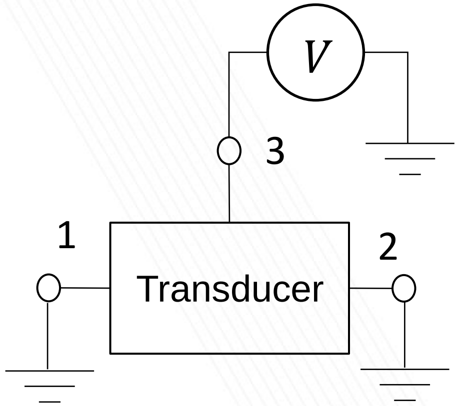
- Substitute in boundary conditions and rearrange in **scattering matrix** form:

$$\begin{bmatrix} B_1 \\ B_2 \\ B_v \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_v \end{bmatrix}$$

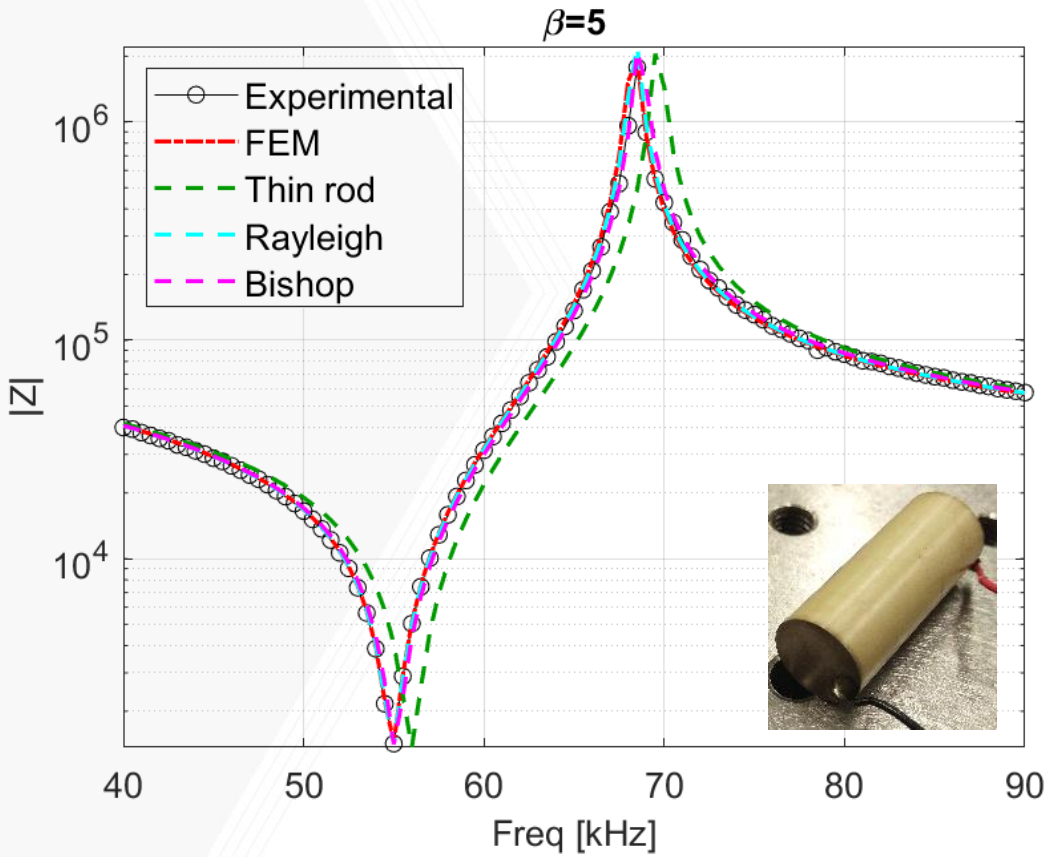




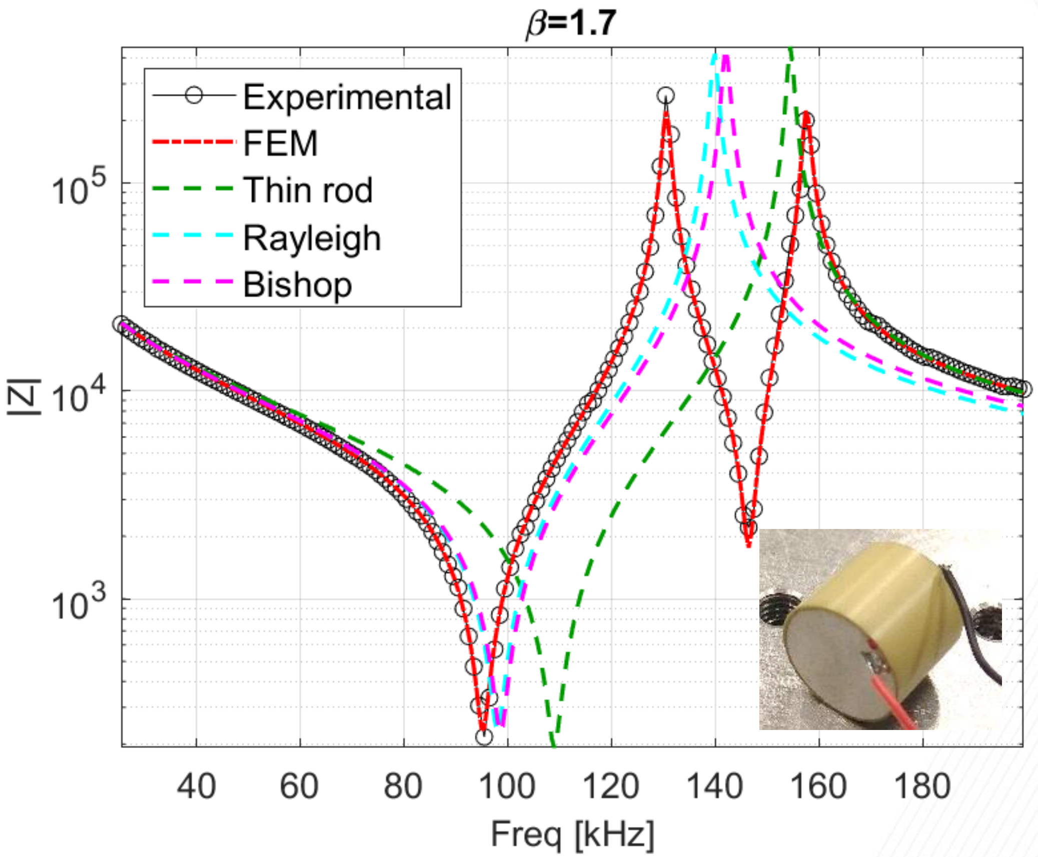
Case studies & comparison to experiments



Electromechanical impedance (in air)

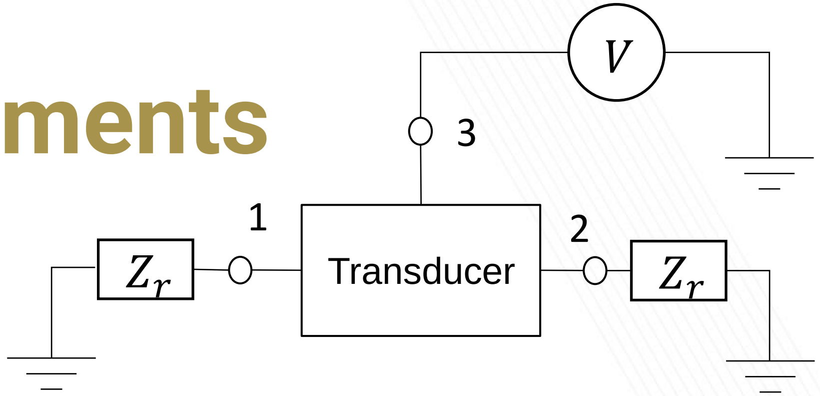


$h = 25mm, a = 5mm$

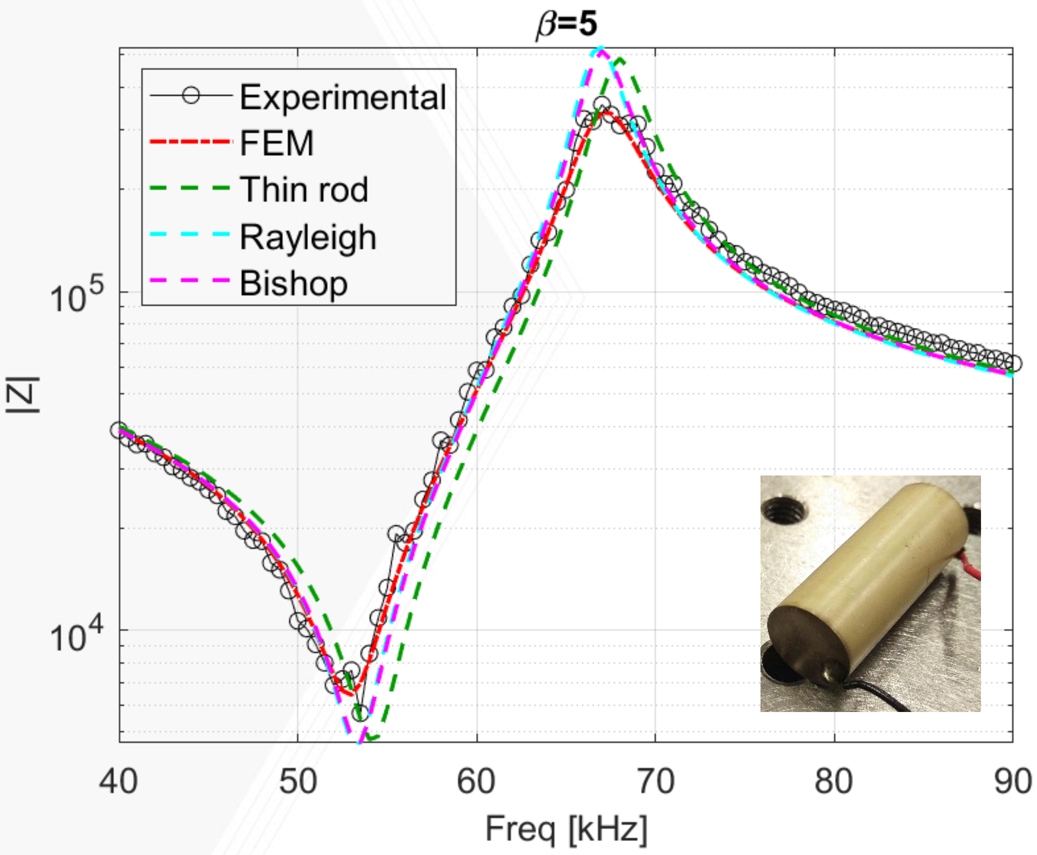


$h = 12mm, a = 7mm$

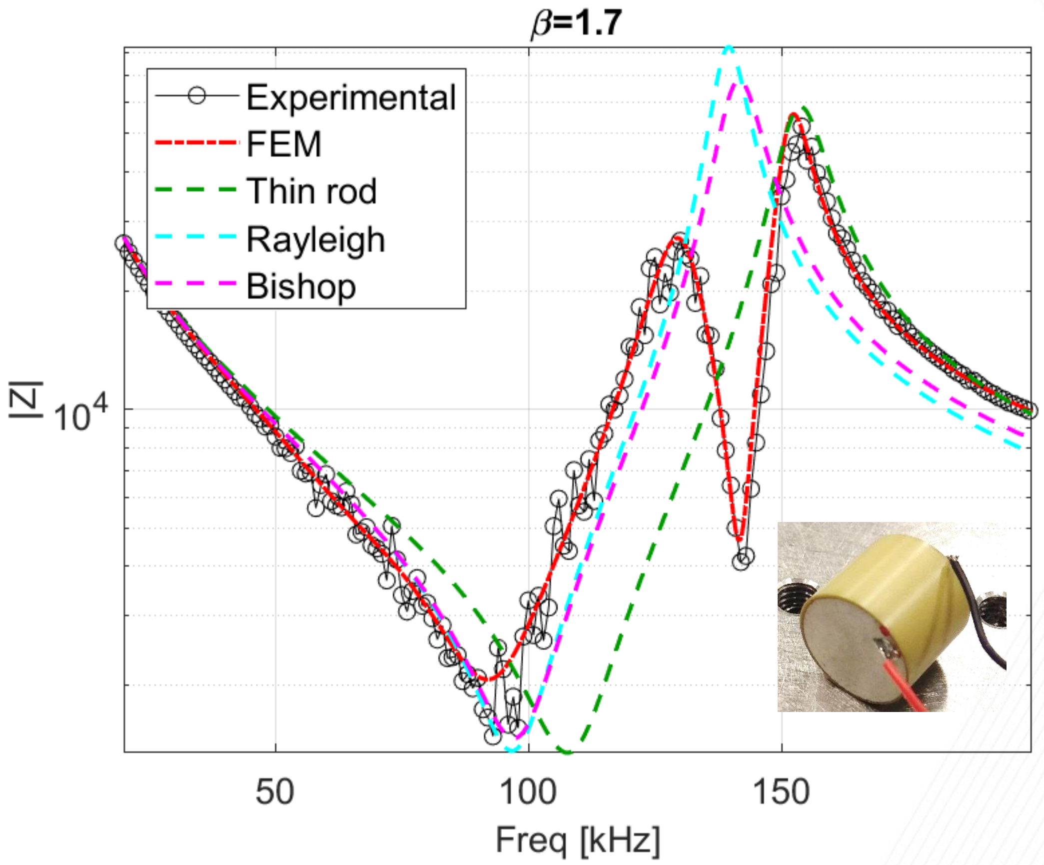
Case studies & comparison to experiments



Electromechanical impedance (in fluid)

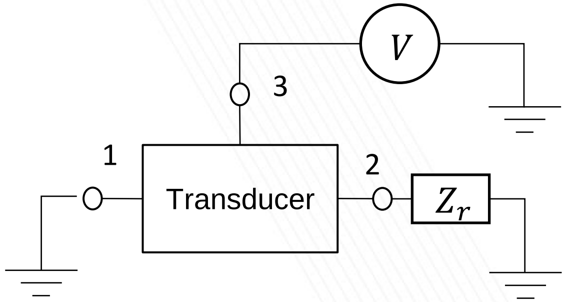


$h = 25mm, a = 5mm$

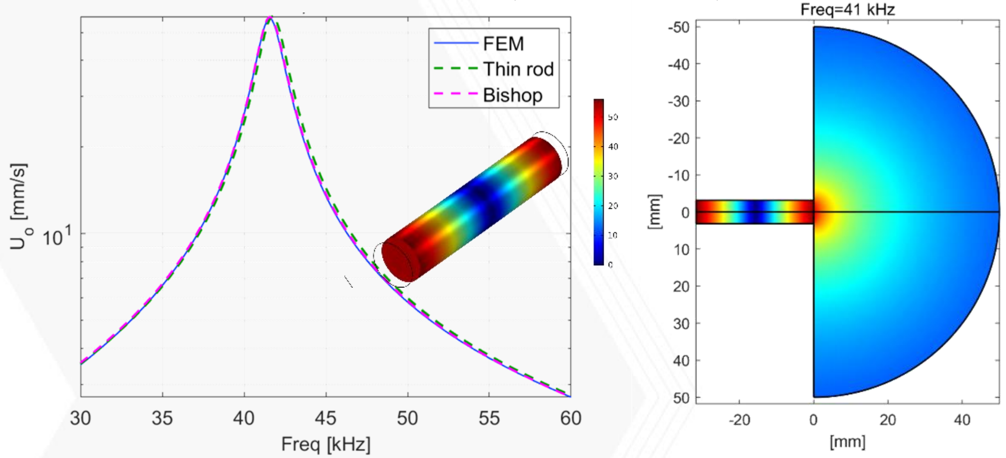


$h = 12mm, a = 7mm$

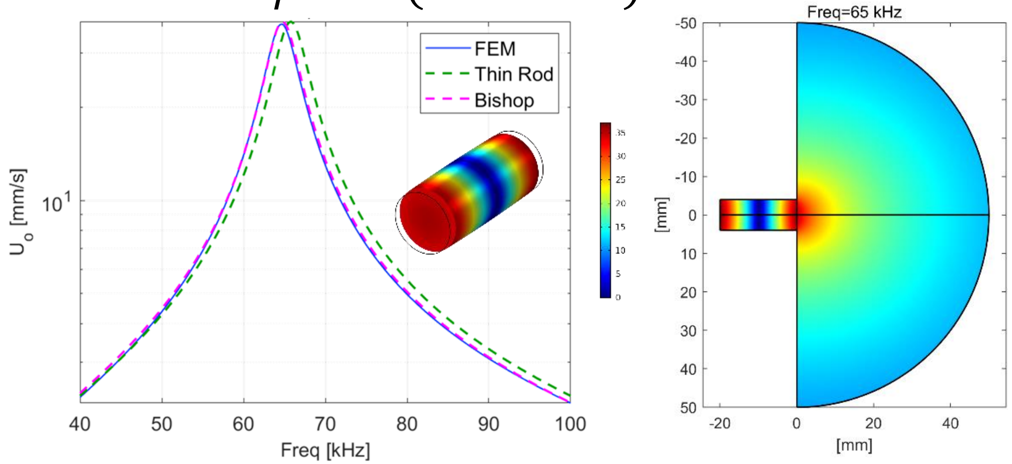
Acoustic wave field (baffled transmitter)

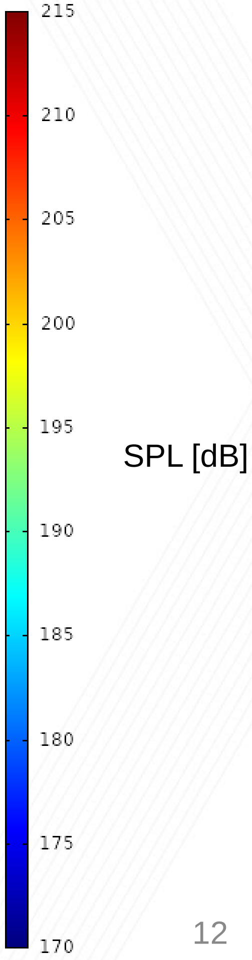


$\beta = 10\ (a \cong 3mm)$

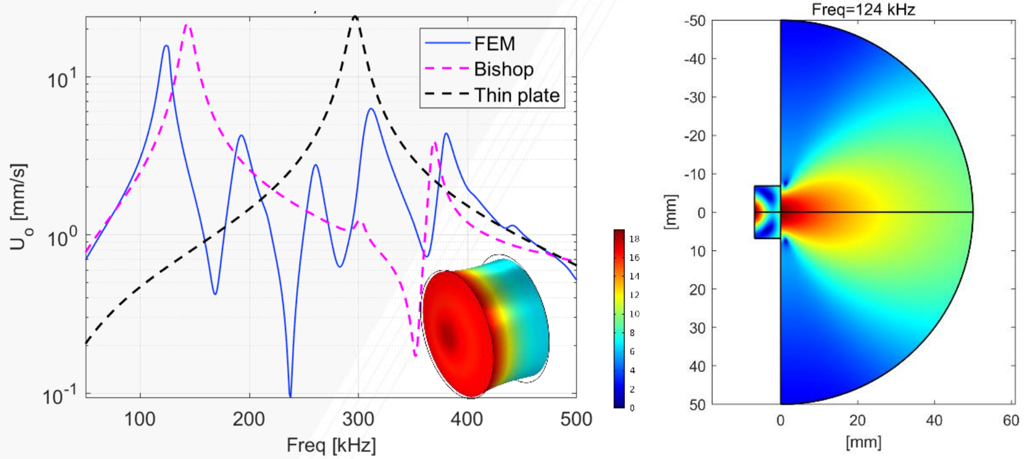


$\beta = 5\ (a \cong 4mm)$

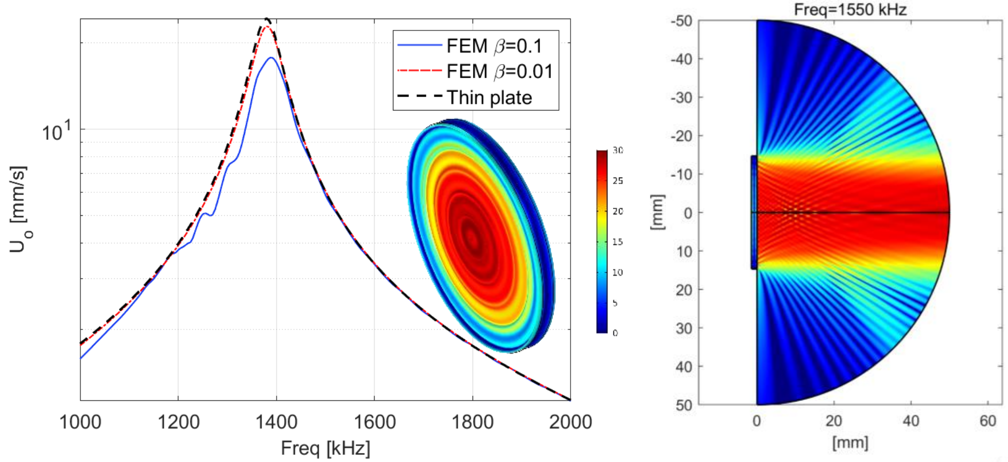




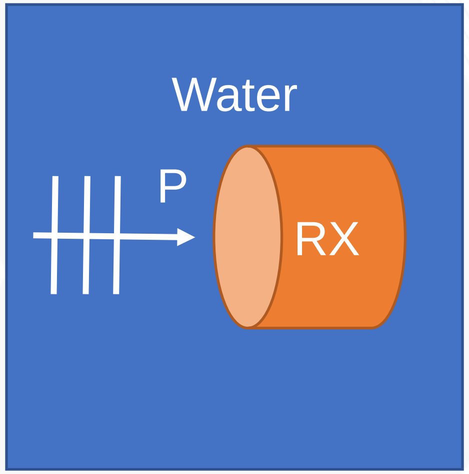
$\beta = 1\ (a \cong 7mm)$

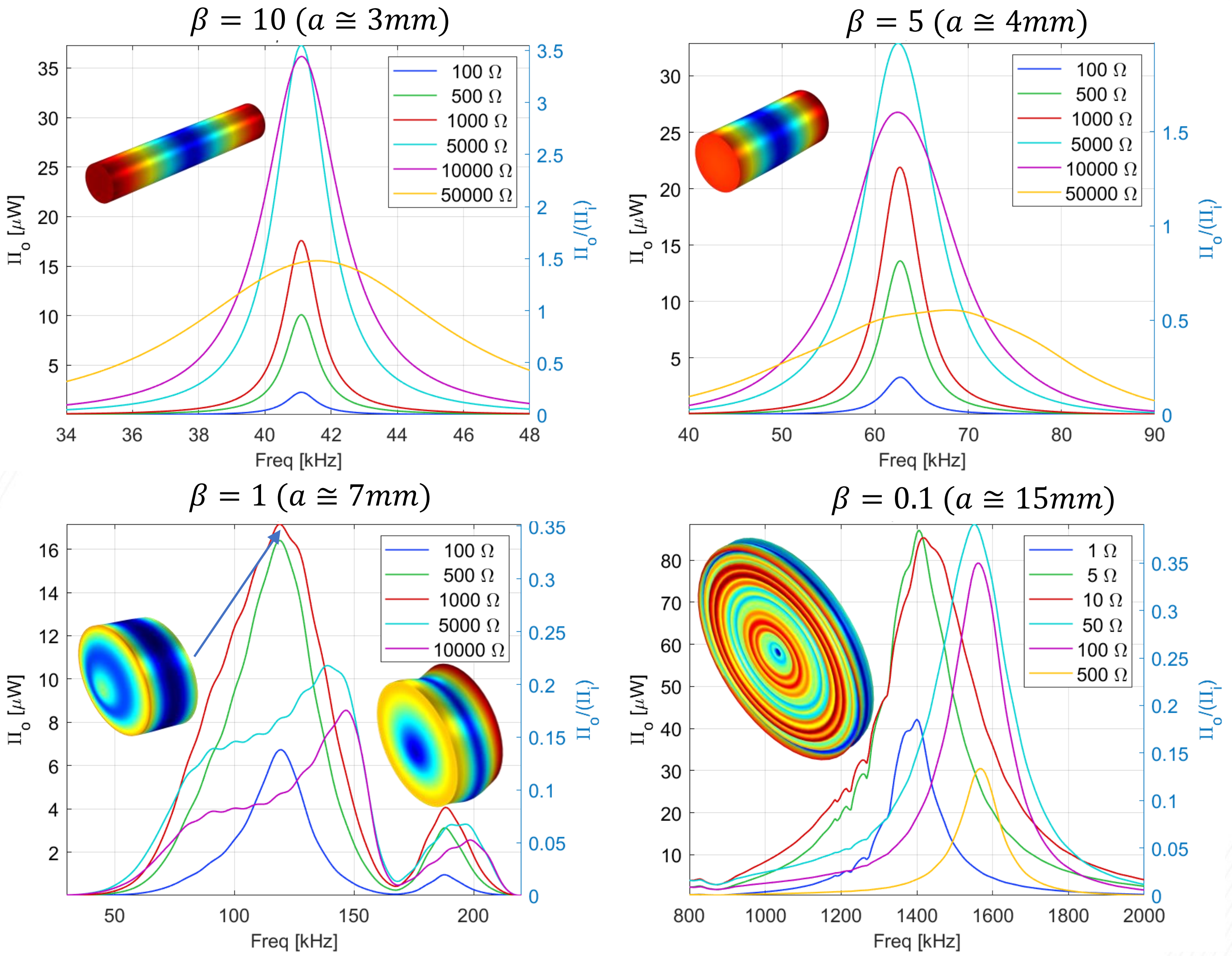


$\beta = 0.1\ (a \cong 15mm)$



Electrical power frequency response (for RX)





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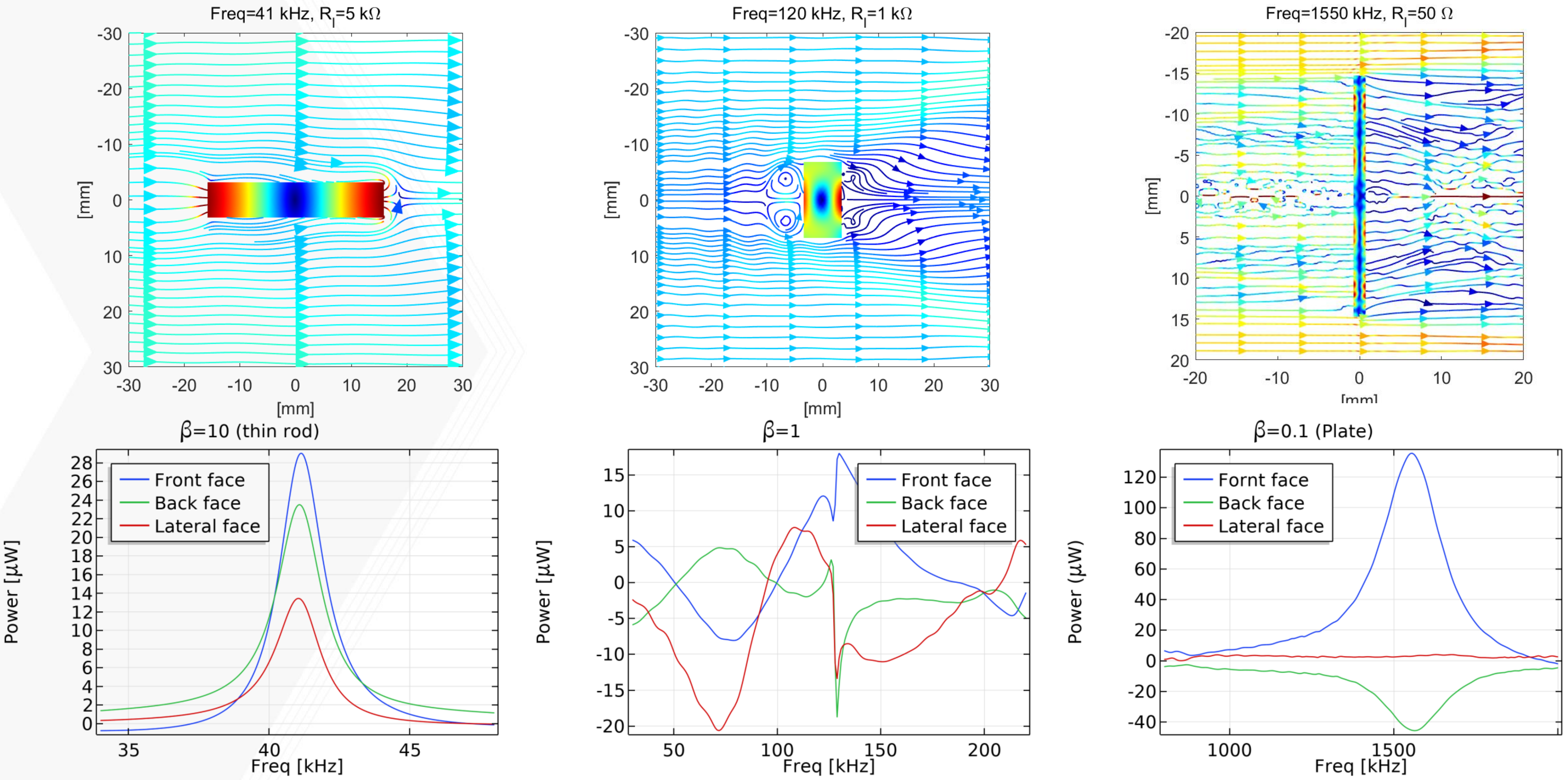
Further investigation of acoustic power flow



Summary and conclusion

- Various theories for analytical modeling of thickness vibration are examined and compared to predict the frequency response.
- Enhanced models (e.g. Rayleigh, Bishop) can be used to correct for baffled transmitters response with aspect ratio $\beta > 3$.
- Analytical models analyzed cannot be used for high accuracy to predict the power output from a moderate aspect ratio transducer (finite-element analysis may become necessary).
- Power output analysis is also conducted along with an account of directivity and power flow.

Acknowledgment

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Thanks!..
Questions?